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CALCULATION OF THE SECONDARY FLOW IN TURBULENT FLUID FLOW

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ABSTRACT. Secondary flow occurs in turbulent fluid flows in direct channels of non-circular cross section in a plane perpendicular to the direction of the main flow. Secondary flow exerts a significant influence on transfer processes in turbulent flows. The article is devoted to an examination of problems associated with secondary flow calculation. The numerical solution of the problem is based on the Reynolds equation.

Fluid motion in turbulent fluid flows in straight channels of non-circular cross section which occurs in a plane perpendicular to the direction of main flow is called secondary flow. The existence of secondary flows in rectangular channels was detected in the experiments of Nikuradze [1] and was subsequently confirmed during investigations by other authors [2-6]. /166*

Secondary flow exerts a significant influence on transfer processes in turbulent flows and it is this factor which determines the ever-increasing interest in the phenomenon under consideration. The study of secondary flow is however limited by the framework of experimental investigation. It is only in the works [3,4] that the question has been posed concerning the description of phenomena by equations which assume the possibility of conducting calculations and imparting appropriate direction to experimental investigation.

This article is devoted to an examination of problems associated with the calculation of secondary flow. A plan for the numerical solution of the problem is proposed. The basis of the calculations derived consists of the Reynolds equation [3] and generalized relationships on the distribution of turbulent intensity in a channel section [7].

1. We shall examine the problem of viscous fluid flow in a channel of square section. Let the x and y coordinate axes be arranged as shown in Figure 1, while the z axis is directed along the channel axis. In a case when the flow characteristic does not change in the direction of motion of the main flow, i.e., of the z axis, fluid motion in the x, y plane may, in agreement with [3], be described by the following equations:

$$U\frac{\partial\Omega}{\partial x}+V\frac{\partial\Omega}{\partial y}-\nu\left(\frac{\partial^2\Omega}{\partial x^2}+\frac{\partial^2\Omega}{\partial y^2}\right)=\frac{\partial^2}{\partial x\partial y}(u^2-v^2)+\left(\frac{\partial^2}{\partial x^2}-\frac{\partial^2}{\partial y^2}\right)uv \tag{1.1}$$

*Numbers in the margin indicate foreign page pagination.

$$\Omega = \frac{\partial U}{\partial y} - \frac{\partial V}{\partial x}, \qquad \frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} = 0. \tag{1.2}$$

Here $\sqrt{u^2}$, $\sqrt{v^2}$ are turbulent intensity components along the x, y coordinate axes; U, V are velocity components of the secondary flow. Along the flow boundary U = V = 0.

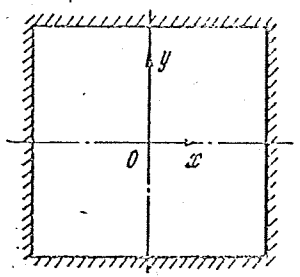

Figure 1.

The problem of the calculation of secondary flow is a closed one if the channel section distribution of correlation functions u^2 - v^2 and uv is known.

2. According to the data in [3] and [4] it is possible to evaluate the expression

$$\alpha = \left(\frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial y^2}\right) uv, \qquad \beta = \frac{\partial^2}{\partial x \partial y}(u^2 - v^2).$$

The relationship $\alpha \ll \beta$ is valid over practically the entire zone of flow. The problem is simplified if we accept $\alpha = 0$. Further if we introduce the dimensionless variables

$$U^* = \frac{U}{W_0}, \quad V^* = \frac{V}{W_0}, \quad x^* = \frac{x}{m}, \quad y^* = \frac{y}{m}$$

where 2m is the side of the square, and W_0 is the maximum value of the flow velocity component along the z axis, the equation (1.1) in this connection will be reduced to the form

$$U\frac{\partial \Omega}{\partial x} + V\frac{\partial \Omega}{\partial y} - \frac{1}{R}\left(\frac{\partial^2 \Omega}{\partial x^2} + \frac{\partial^2 \Omega}{\partial y^2}\right) = \frac{\partial^2 F}{\partial x \partial y}, \quad F = u^2 - v^2. \tag{2.1}$$

(Here U, V, Ω, etc. are functions which are already dimensionless).

In agreement with the known approach, the relationships (1.2) are transformed as follows:

$$-\Omega = \frac{\partial^2 \Psi}{\partial x^2} + \frac{\partial^2 \Psi}{\partial y^2}, \quad U = -\frac{\partial \Psi}{\partial y}, \quad V = \frac{\partial \Psi}{\partial x} \tag{2.2}$$

On the basis of a generalization of experimental data for channels of non-circular cross section the dependency was obtained in [7] of velocity pulsation intensity on the motion cycle and on the coordinates. In agreement with these results the function F(x,y) has the form /167

$$F = N\left[e^{(ax+by)} - e^{(cx+dy)}\right], \qquad N = \varkappa(1 - W_m)^2 W^2 \tag{2.3}$$

where W_m is dimensionless average velocity along the channel section,

$$\varkappa = 0.0342, \quad a = 2.34, \quad b = -0.8, \quad c = 1.54, \quad d = 0.$$

Velocity distribution W(x,y), which enters into the determination of function F(x,y), may be determined by the method in [8].

The relationships (2.1), (2.2) and (2.3) were made the basis of calculations of secondary flow in a turbulent fluid flow in a channel of square section.

3. The problem was solved by the difference method. The calculation region $0 \leq x \leq 1$, $0 \leq y \leq 1$ was covered with a coordinate grid of lines x_i and y_k ($i = 1, \ldots, n$, $k = 1, \ldots, n$) with step h. A difference analog was constructed for the system (2.1), (2.2) by means of replacing the differential operators with different operators.

The difference analog of the equation (2.1) is

$$-a_{ik}\Omega_{i-1k} - c_{ik}\Omega_{i+1k} - b_{ik}\Omega_{ik-1} - d_{ik}\Omega_{ik+1} + e_{ik}\Omega_{ik} = f_{ik}$$

$$a_{ik} = \frac{A}{h^2} + R\frac{U_{ik}}{h}, \quad c_{ik} = \frac{A}{h^2}, \quad A = \frac{1}{1 + 0.5RhU_{ik}} \quad (U_{ik} \geqslant 0)$$

$$a_{ik} = \frac{A}{h^2}, \quad c_{ik} = \frac{A}{h^2} - R\frac{U_{ik}}{h}; \quad A = \frac{1}{1 - 0.5RhU_{ik}} \quad (U_{ik} < 0).$$

Similar expressions occur for b_{ik} and d_{ik}

$$e_{ik} = a_{ik} + c_{ik} + b_{ik} + d_{ik}.$$

In approximation on a 5-point model, usually employed in these cases, was used for (2.2).

The base of the solution plan consisted of a stabilizing correction plan [9] with operators

$$\Lambda'\Omega = e'_{ik}\Omega_{ik} - a_{ik}\Omega_{i-1k} - b_{ik}\Omega_{ik-1}$$

$$\Lambda''\Omega = e''_{ik}\Omega_{ik} - c_{ik}\Omega_{i+1k} - d_{ik}\Omega_{ik+1}$$

The solution plan in its operation form is

$$(\sigma_1 + e'_{1ik})\,\Omega^{l+1/2}_{ik} - a_{1ik}\,\Omega^{l+1/2}_{i-1,\,k} - b_{1ik}\,\Omega^{l+1/2}_{i,\,k-1} = (\sigma_1 - e''_{1ik})\,\Omega^{l}_{ik} + c_{1ik}\,\Omega^{l}_{i+1,\,k} + d_{1ik}\,\Omega^{l}_{i,\,k+1} - g_{ik}\,\psi^{l}_{1k} + f_{1ik}$$

$$(\sigma_1 + e''_{1ik})\,\Omega^{l+1}_{ik} = \sigma_1\Omega^{l+1/2}_{ik} + c''_{1ik}\,\Omega^{l}_{ik} + c_{1ik}\,(\Omega^{l+1}_{i+1,\,k} - \Omega^{l}_{i+1,\,k}) + d_{1ik}\,(\Omega^{l+1}_{i,\,k+1} - \Omega^{l}_{i,\,k+1}) - g_{ik}\,(\psi^{l+1}_{ik} + \psi^{l}_{ik})$$

$$(\sigma_2 + e'_{2ik})\,\psi_{ik}^{l+1/2} - a_{2ik}\psi_{i-1,k}^{l+1/2} - b_{2ik}\psi_{i,k-1}^{l+1/2} = (\sigma_2 - e''_{2ik})\,\psi_{ik}^{l} + \\ + c_{2ik}\psi_{i+1,k}^{l} + d_{2ik}\psi_{i,k+1}^{l} + \Omega_{ik}^{l+1/2} \qquad (3.1)$$

$$(\sigma_2 + e''_{2ik})\,\psi_{ik}^{l+1} = \sigma_2\psi_{ik}^{l+1/2} + e''_{2ik}\psi_{ik}^{l} + c_{2ik}(\psi_{i+1,k}^{l+1} - \psi_{i+1,k}^{l}) + d_{2ik}(\psi_{i,k+1}^{l+1} - \psi_{i,k+1}^{l})$$

$$(i = 1, \ldots, n; \quad k = 1, \ldots, n)$$

$$e_{ik}' \geqslant a_{ik} + b_{ik}, \qquad e_{ik}'' \geqslant c_{ik} + d_{ik}, \qquad e_{ik}' + e_{ik}'' = e_{ik}$$

$$g_{ik} = \frac{2}{h}(c_{1ik}\delta_i^n + d_{1ik}\delta_k^n), \qquad \delta_m^n = \begin{cases} 1, & m = n \\ 0, & m \neq n \end{cases}.$$

In the presentation for the vector

$$\Phi = \begin{pmatrix} \Omega \\ \Psi \end{pmatrix}$$

the solution plan is written as follows: /168

$$(\Sigma + E')\,\Phi_{ik}^{l+1/2} - A\Phi_{i-1,k}^{l+1/2} - B\Phi_{i,k-1}^{l+1/2} = (\Sigma - E'')\,\Phi_{ik}^{l} + C\Phi_{i+1,k}^{l} + D\Phi_{i,k+1}^{l} + F$$
$$(\Sigma + E'')\,\Phi_{i,k}^{l+1} = \Sigma\Phi_{ik}^{l+1/2} + E''\Phi_{ik}^{l} + C(\Phi_{i+1,k}^{l+1} - \Phi_{i+1,k}^{l}) + D(\Phi_{i,k+1}^{l+1} - \Phi_{i,k+1}^{l}).$$

The following designations are introduced for solution plan notation

$$A = \begin{pmatrix} a_{1ik} & 0 \\ 0 & a_{2ik} \end{pmatrix}, \quad B = \begin{pmatrix} b_{1ik} & 0 \\ 0 & b_{2ik} \end{pmatrix}, \quad C = \begin{pmatrix} c_{1ik} & 0 \\ 0 & c_{2ik} \end{pmatrix}, \quad D = \begin{pmatrix} d_{1ik} & 0 \\ 0 & d_{2ik} \end{pmatrix}$$
$$E' = \begin{pmatrix} e'_{1ik} & 0 \\ -1 & e'_{2ik} \end{pmatrix}, \quad E'' = \begin{pmatrix} e''_{1ik} & g \\ 0 & e''_{2ik} \end{pmatrix}, \quad \Sigma = \begin{pmatrix} \sigma_1 & 0 \\ 0 & \sigma_2 \end{pmatrix}, \quad F = \begin{pmatrix} f_{1ik} \\ f_{2ik} \end{pmatrix}.$$

Here we shall introduce data on the convergence rate of the iteration process (3.1). In calculations the matrix of coefficients Λ was symmetrical, $0 \leq x \leq 1$, $0 \leq y \leq 1$, and the interval [0,1] was divided into n parts. The iteration result was the fulfillment of condition

$$\max_{i,k}\left|\frac{\psi_{ik}^{l} - \psi_{ik}^{l-1}}{\psi_{ik}^{l}}\right| < \varepsilon = 0.001, \qquad l = l^*.$$

Further, the following was accepted

$$\psi_{ik}^{0} = \Omega_{ik}^{0} = 0, \quad f_{1ik} = 1000, \quad f_{2ik} = 0, \quad \sigma_1 = \sigma_2 = \sigma$$
$$n = 10, \quad l^* = 32 \quad (\sigma = 50), \qquad n = 20, \quad l^* = 46 \quad (\sigma = 110).$$

In the case g = 0 (the function Ω equals 0 along the entire boundary) the following evaluations are obtained:

$$n = 10, \quad l^* = 38 \quad (\sigma = 70), \qquad n = 20, \quad l^* = 52 \quad (\sigma = 140).$$

4. It follows from the results of the solution of this problem that a secondary flow does in fact occur in the sectional plane of a square channel; the flow represents a closed fluid motion in each of the octants of the square. The results of calculation with Reynolds number R = 50,000 are shown in Figures 2, 3, 4.

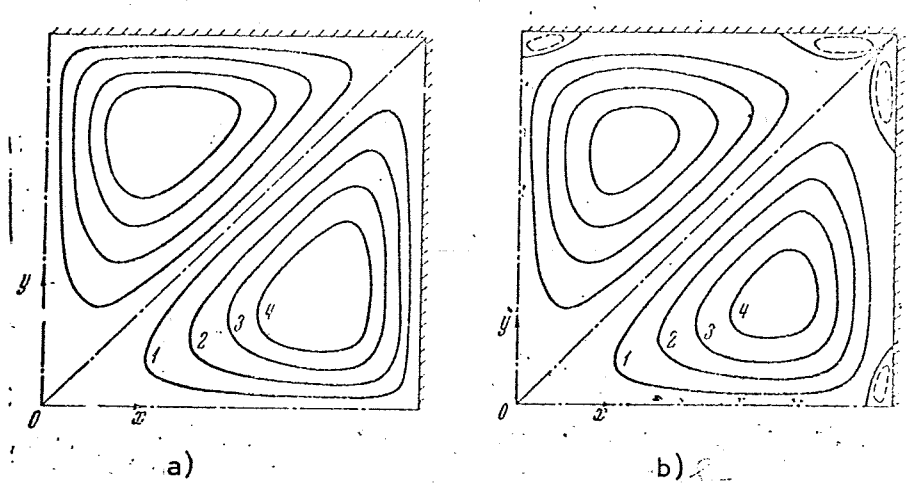

Figure 2.

The value W = 1 was used in preliminary calculations for the function W(x,y), which enters into equations (2.3) and (2.4). A comparison of the results of calculation obtained with such a determination of W(x,y) (here the functions u^2 and v^2 monotonically increase in the direction of the channel wall) and according to the field W(x,y), calculated in accordance with the technique in [8] (here a maximum near the wall occurs for the function u^2 and v^2), reveals that a difference in the corresponding results occurs only in the region directly adjacent to the wall.

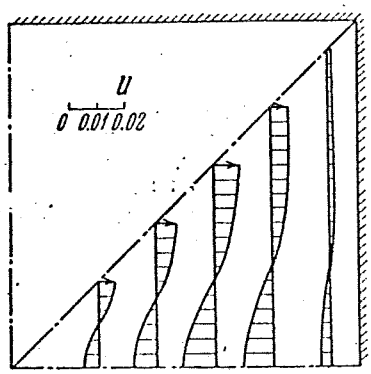

Figure 3.

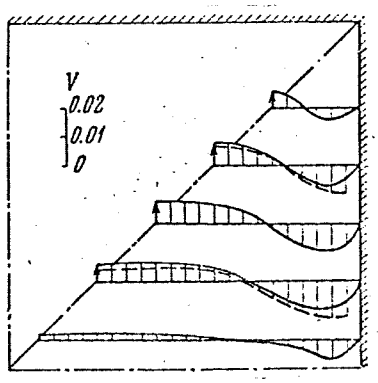

Figure 4.

In order to investigate the influence of the nature of the behavior of functions u^2 and v^2 in direct proximity to the wall on the secondary flow characteristic the following determination for function N in (2.3) was utilized in calculations: /169

$$N = \gamma \left(\frac{s+1}{s}\right)^2 (1 - x^s)(1 - y^s) \qquad \left(\gamma = 0.0036 + \frac{60}{R}\right). \tag{4.1}$$

Figure 2 shows the flow pattern in the x,y plane in the form of streamlines Ψ = const ($\psi_1 = 0.44 \cdot 10^{-3}$, $\psi_2 = 0.88 \cdot 10^{-3}$, $\psi_3 = 1.32 \cdot 10^{-3}$, $\psi_4 = 1.76 \cdot 10^{-3}$) and for two values of the parameter s in (4.1): s = 30 (corresponds to a monotonic increase in functions u^2 and v^2 from the channel center to the last calculation point near the wall) and s = 10 (the nature of the behavior of functions u^2 and v^2 in the zone near the wall agree with the results of work [10]).

The results revealed that in the first case the formation of one secondary vortex occurs in the unit cell of the channel (Figure 2a). In the second case (Figure 2b) there is an additional formation of two weak vortices in the area adjacent to the wall. In contemporary investigations there are no experimental data concerning the existence of such a flow structure in the zone near the wall. In the published works [2-5], data on secondary flows in the zone near the wall are not introduced and corresponding questions are not discussed. A final solution of this problem through calculation may be obtained with experimental clarification of the nature of the behavior of functions u^2 and v^2 in the zone near the wall.

Calculated curves of velocity components U and V for the secondary flow in an octant of the square channel are shown in Figures 3 and 4; Figure 4 also shows experimental curves (dashed curves) of the component V in accordance with the [4] cited.

Close agreement between the results of calculation and measurement permit us to conclude that in the statement of the problems examined basic factors are considered which lead to the appearance and existence of secondary flows.

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